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Application of the Global Positioning System in Determination of Vehicular Acceleration

Shane D. Pinder,* Trever G. Crowe,† and Peter N. Nikiforuk‡
University of Saskatchewan, Saskatoon, Saskatchewan S7N 5A9, Canada

Aircraft landing and takeoff performance monitoring is an area of research aimed at improving the information available to the pilot for decision making during takeoff or landing. A system capable of instantaneously determining the stopping distance of an aircraft could form an integral component of a monitoring system. Particularly difficult to quantify is the frictional coefficient between the runway and the aircraft tires, should such a measurement be necessary. In secluded far-northern regions, where a monitoring system would be particularly useful given adverse weather, few airports are equipped to attempt frictional measurements. In such instances a monitoring system would need to be totally self-contained and able to determine aircraft ground speed, acceleration, and position relative to the end of the runway. Prediction of the aircraft's location at rest is then possible. It is proposed that the Global Positioning System (GPS) be used to determine aircraft acceleration, ground speed, and position relative to the end of the runway. A practical evaluation of the feasibility of this proposal showed clear superiority of a GPS-derived acceleration over a more traditional method employing accelerometers. Advantages of the GPS-derived measurement include a modest noise level, insusceptibility to gravity and temperature-influenced variations, and far simplified mounting criteria.

Nomenclature

a	=	instantaneous magnitude of deceleration of a vehicle
a_x	=	component of vehicular acceleration in the x direction
a^*	=	raw measurement from an accelerometer
dt	=	discrete time step in a Kalman filter
g_x	=	component of gravity in the x direction
s	=	instantaneous required stopping distance of a vehicle
V_1	=	critical engine failure recognition speed for an aircraft
v	=	instantaneous forward speed of a vehicle
v_k	=	measurement noise in a Kalman filter
w_k	=	process noise in a Kalman filter
x	=	one-dimensional position of a vehicle measured with respect to a datum
y_k	=	measured speed of a vehicle derived from a GPS receiver
Δa	=	deceleration uncertainty
Δs	=	stopping distance uncertainty
Δv	=	speed uncertainty

Aircraft Landing and Takeoff Performance Monitoring

UNLIKE instrument landing systems (ILS), which rely on precise positioning to guide the aircraft to touchdown, landing and takeoff performance monitoring systems are aimed at averting runway overrun. In northern regions this has been identified[1] as a common problem. Typical causes of runway overrun include engine failure on takeoff and reduced braking resulting from runway contamination.

The critical engine failure recognition speed V_1 is defined as the speed above which takeoff could continue safely if the most critical engine failed.[2] V_1 is often calculated prior to startup based on aircraft parameters and estimation of runway and weather conditions. Choosing a throttle setting to reach V_1 is a more complicated matter. If a low-throttle setting is chosen, takeoff rejection initiated at a speed slightly below V_1 can result in runway overrun. If a high-power setting is chosen, other problems arise. Engine life depends largely on its peak power setting. As well, the likelihood of an engine failure on takeoff increases with increased power setting.

Operators often use the so-called balanced field concept to calculate the lowest possible power setting for use during takeoff. Then, at speeds below V_1 there is always enough runway remaining to abort takeoff. Once V_1 is reached, the aircraft could safely takeoff even in the event of the failure of one engine. With this in mind, V_1 becomes a "decision" speed. Figure 1 shows this scenario with a takeoff rejection initiated at a decision speed of 80 m/s on a 2400-m runway.

Performance monitoring systems[3] that provide similar information are currently in existence for use during takeoff but are seldom used. In such systems the pilot is required to provide runway length information as well as runway frictional coefficient data based on measurements provided by ground-based observers.

A similar system for use during approach and landing is currently unavailable because of the inability for the pilot to provide remaining runway length. It is proposed that runway length information be measured independently by way of precise positioning from a global positioning system (GPS) receiver. With this innovation the same observer system could be used for both takeoff and landing.

Far-Northern Environment

The runway overrun problem is further aggravated in inclement weather where runway surfaces are contaminated by water or ice. Far-northern regions experience this sort of climate over six months of the year. Further, as such regions are relatively less populated, facilities may receive infrequent maintenance. These factors contribute to the difficulty that pilots experience.

Many airports in far-northern regions are gravel surfaced. The behavior of a gravel runway can be unpredictable, especially when temperatures are near the freezing point. Measurements of runway friction attempted in such conditions would be relatively unreliable.

The availability of radio navigation systems in far-northern regions is also an issue. Although such facilities exist, they are sparsely distributed and tend to service the airports of major population centres. Air carriers that service airports in support of mining and forestry are less likely to have reliable access to radio navigation facilities. The GPS has provided some relief to this problem.

GPS

The GPS is a satellite navigation system that provides a means of calculating time, position, and velocity data using coded signals,

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*Doctoral Candidate, College of Engineering.
†Associate Professor, College of Engineering.
‡Dean Emeritus, College of Engineering.

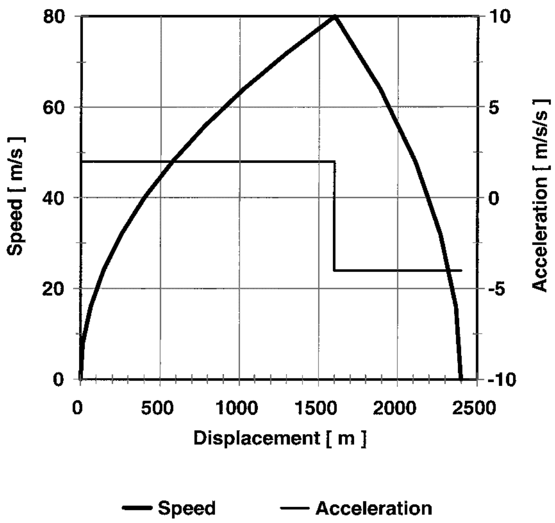

Fig. 1 "Balanced field concept" is used to determine the maximum speed at which a takeoff can be safely aborted.

which can be processed using a receiver.[4] A minimum of four satellite signals are used to compute three-dimensional positions. A GPS receiver derives position information by measuring the time required for a signal to be transmitted from a satellite with a known position. There are several sources of inaccuracy in this process including receiver noise, tropospheric delay, multipath error, satellite clock errors, orbit errors, and ionospheric delay. Until 1 May 2000 the U.S. Department of Defense injected intentional signal degradation or selective availability (SA) for security reasons. At the time of this investigation, SA was by far the largest contribution to position error, on the order of 100 m. However, this error can be described as a slow wandering bias error. The resulting velocity error from time differentiation was less than 1 m/s. Further, the velocity error changed slowly resulting in a virtually negligible acceleration error. The other, smaller errors were unavoidable, but most represented relatively steady bias errors. With the exception of receiver noise and multipath, these errors were highly repeatable when considering time intervals of less than 1 s.

Acceleration from GPS

The notion of acquiring a measurement of acceleration from GPS is not new. When compared to the measurement obtained from an accelerometer, a GPS-derived measurement of acceleration can be used to determine the gravity vector. This technique has been used in airborne gravimetry to determine the gravitational constant with accuracies[5] on the order of 10^{-5} m/s^2, but requires a substantial amount of data to filter out vibrational disturbances. More recently, it has been proposed that a GPS-derived measurement of acceleration together with an accelerometer could yield a representation of the gravity vector[6] to be used as an attitude reference. Such an application would require a real-time GPS-derived measurement of acceleration if used on vehicles with rapidly changing attitude.

Although accelerometers have been historically used to determine aircraft acceleration, it is impossible to remove the significant and adverse influence of the gravity vector without additional instrumentation to accurately measure aircraft attitude. Accelerometers do not respond only to acceleration, but rather the force per unit mass on an element of known mass. With reference to Fig. 2,

$$a^* = a_x + g_x \tag{1}$$

From the foregoing, it can be shown that for small angles the gravity vector introduces an error of 0.171 m/s^2 per degree of inclination. This problem is avoided through the use of a GPS-derived measurement. This is an especially appropriate choice given the need to locate the aircraft with respect to the end of the runway, an application in which GPS is well employed.

In aircraft landing and takeoff performance monitoring the desired acceleration measurement should reflect the overall vehicular

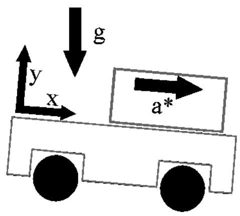

Fig. 2 "Accelerometer gravity error" results from the accelerometer measuring a component of the gravity vector that cannot be determined without an accurate measurement of the sensor inclination with respect to horizontal.

acceleration as opposed to vibration of subcomponents. Accelerometers are well suited to measurement of vibration, where the influence of gravity need not be removed from the measurement, but a GPS-based measurement is clearly superior in stable, piecewise constant vehicular acceleration.

By virtue of the nature of the GPS signal, an accurate acceleration measurement can be obtained with the use of a single GPS receiver. As a result, a performance monitoring system could be designed in the absence of differential corrections, which may be unavailable in far-northern regions where performance monitoring is most needed. This application of GPS without differential corrections appears to be novel.

Required Accuracy

The level of uncertainty in a GPS-derived measurement of acceleration depends on the type of filter used to remove noise caused by differentiation. In any case the amount of error will not depend on the magnitude of the instantaneous signal. Rather, the standard deviation of noise should depend primarily on the number of satellites in view.

The equation governing instantaneous stopping distance of a vehicle is quite simple:

$$s = v^2/2a \tag{2}$$

Uncertainty analysis provides

$$\Delta s = \Delta v \frac{\partial s}{\partial v} + \Delta a \frac{\partial s}{\partial a} \tag{3}$$

$$= \Delta v \frac{v}{a} - \Delta a \frac{v^2}{2a^2} \tag{4}$$

Rearranging Eq. (2) to solve for deceleration, substituting into Eq. (4), and solving for deceleration uncertainty gives

$$\Delta a = \Delta v(v/s) - \Delta s(v^2/2s^2) \tag{5}$$

Instances where deceleration uncertainty has the most impact occur when the forward speed is high and when the rearward acceleration is low. To establish an acceptable level of uncertainty in the measurement of deceleration, consider a modest V_1 on a long runway. Runways are seldom longer than 4000 m. Over half of the length of the runway would be required to reach V_1, and so assume $s = 2000$ m and $v = V_1 = 90$ m/s.

The margin of safety in the stopping distance would need to be larger than the length of the aircraft so that the uncertainty in the estimation of stopping distance can be conservatively chosen as $\Delta s = 150$ m. From GPS a typical speed uncertainty determined using constant speed trials is $\Delta v = 1.12$ m/s.

Substituting these values into Eq. (5) yields a conservative maximum acceptable uncertainty in the measurement of deceleration:

$$\Delta a = 0.101 \text{ m/s}^2 \tag{6}$$

Although a lower speed can apply to many aircraft, the runway length should be considered very conservative. In many cases higher uncertainty may be acceptable.

Experimental Investigation

A NovAtel 3151RE GPS receiver capable of collecting pseudorange measurements at a rate of 20 Hz was selected for use in the test apparatus. The receiver logged position and velocity at 10 Hz. The velocity measurement from the GPS receiver in the test apparatus was derived from time differentiation of position or carrier

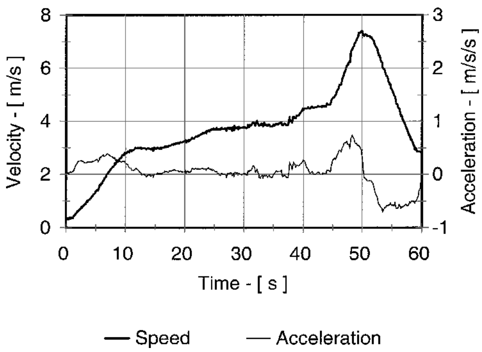

Fig. 3 Speed and acceleration of a test vehicle derived from GPS data in the presence of SA.

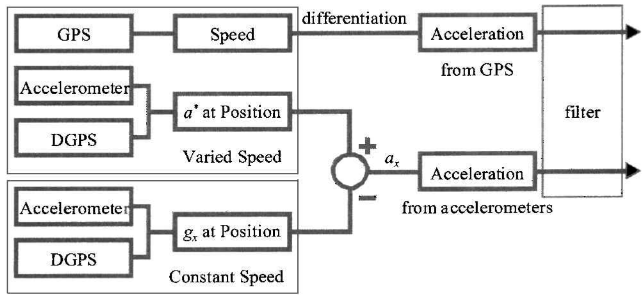

Fig. 4 Block diagram of data-processing technique.

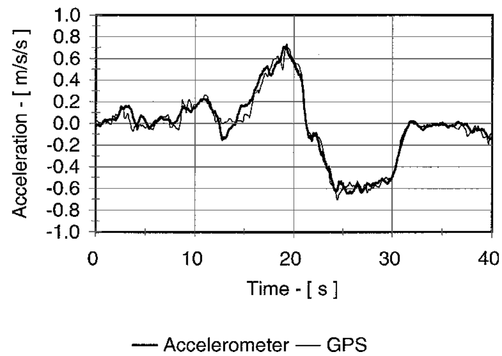

Fig. 5 Comparison of GPS-derived and accelerometer-derived measurements of the acceleration of a test vehicle.

phase Doppler measurements owing to the manufacturer's proprietary algorithm, and the acceleration measurement was a filtered time differentiation of this velocity measurement, obtained using a Kalman filter. In this Kalman filter the third derivative of position is considered to be a random process. This is of course untrue but is a reasonable approximation of the dynamics.

In state-space form the system dynamics

$$\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix}_{(k+1)} = \begin{bmatrix} 1 & dt & 0 & 0 \\ 0 & 1 & dt & 0 \\ 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix}_{(k)} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_k \end{bmatrix} \tag{7}$$

and the observer

$$y_k = [0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix}_{(k)} + v_k \tag{8}$$

form the foundation of the Kalman filter. This filter requires identification of the variance of the two random variables. The measurement noise is easily approximated during constant speed trials as later discussed. This leaves one variance, that of the process noise, to be chosen.

Figure 3 shows the speed of a vehicle obtained from a GPS receiver in the presence of selective availability and acceleration computed using the Kalman filter. This measurement of acceleration can be compared to a more traditionally obtained measurement.

Testing has been undertaken to verify the accuracy of the acceleration measurement derived from GPS data. The apparatus consisted of a vehicle-mounted GPS receiver, a bank of accelerometers mounted with parallel axes of measurement, and a data acquisition system. The vehicle was rail mounted with no suspension system. Four identical accelerometers provided a confident measure of acceleration. The data acquisition system collected these data at a rate of 20 Hz, electrically synchronized with the GPS receiver's collection of raw pseudorange. During constant-speed trials, the accelerometers were used to determine the slope of the rail surface so that the influence of the gravity vector could be calculated. This slope information was cross matched with geographic location through the use of differential GPS. Twenty constant-speed trials were conducted, yielding a reliable measurement of slope. This method of accounting for rail slope implicitly accounts for any bias errors present in the accelerometers. During trials where the vehicle speed varied, the accelerometer data were corrected for the influence of minor pitch changes by subtracting the known slope at the instantaneous position. Both measurements of acceleration were filtered using the same algorithm. The GPS-derived acceleration measurement was then compared with acceleration data from the bank of accelerometers, after accounting for the effect of gravity. This is shown schematically in Fig. 4.

The use of redundant accelerometers afforded the possibility to obtain increased confidence in the measurement of acceleration. The four accelerometers used in the experimental apparatus agreed with one another very well. The covariance matrix describes how the data collected from each sensor vary with one another. In completely uncorrelated data off-diagonal values would be zero. In completely correlated data all values would be equal. For the collected accelerometer data a covariance matrix

$$\text{cov} = \begin{bmatrix} 1.779 & 1.776 & 1.775 & 1.772 \\ 1.776 & 1.779 & 1.775 & 1.776 \\ 1.775 & 1.775 & 1.779 & 1.773 \\ 1.772 & 1.776 & 1.773 & 1.779 \end{bmatrix} \text{m}^2/\text{s}^4$$

was determined.

It can be concluded that the accelerometer data, while being variable with a standard deviation on the order of 1.33 m/s^2, were highly correlated. This demonstrated that the accelerometer data represented a confident measure of the acceleration of the vehicle component to which the accelerometers were attached.

Figure 5 shows a comparison of the GPS-derived acceleration with that from the accelerometers for one of 10 trials where speed was varied. Other trials yielded similar results. Although both measurements of acceleration were filtered in exactly the same manner, there is clearly no superiority in the accelerometer measurement. It had been expected that, given the care with which the accelerometer data were corrected, the GPS-derived acceleration would be notably time delayed and contain noise. Conversely, it would appear that in those instances where the vehicle acceleration was steady and most similar to what might be expected of a large aircraft the two measurements are in agreement.

Figure 6 shows the calculated difference, for the same trial, between the two measurements of acceleration. This does not represent the error in the GPS-derived measurement, as the accelerometer measurement also lags the "real" acceleration because of filtering. The standard deviation of the difference is 0.054 m/s^2. This falls well within the established conservative maximum uncertainty of 0.101 m/s^2. Closer analysis shows that the calculated difference falls within the maximum uncertainty over 90% of the time.

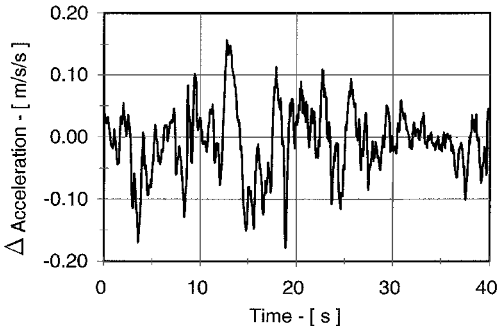

Fig. 6 Difference between GPS-derived and accelerometer-derived measurements of acceleration of a test vehicle were within 0.10 m/s^2, 90% of the time.

With regard to the dynamic range of this investigation, the acceleration and speed associated with aircraft takeoff and landing are typically larger than those investigated. In the investigation speeds in excess of 10 m/s were not experienced, and acceleration was typically 0.5 m/s^2. This difference in dynamic range should have little effect on the accuracy or resolution of the GPS-derived speed measurement, which is governed by jerk. Because it is dependent on the ratio of speed accuracy to the change in speed, the corresponding accuracy of the acceleration measurement should improve at higher accelerations.

Conclusions

It has been demonstrated that the GPS is able to provide a measurement of vehicle speed that is sufficiently reliable to determine acceleration with an uncertainty of under 0.10 m/s^2. This accuracy should be achievable for acceleration in excess of 0.5 m/s^2. Clear advantages in using GPS over the more conventional sensor, an accelerometer, include insusceptibilities to the gravity vector, vibrational disturbances, and temperature fluctuations.

A GPS-derived acceleration together with speed and runway position can be used to determine the instantaneous stopping distance of an aircraft during takeoff or landing. A system capable of performing this operation could form an integral component of a landing and takeoff performance monitor.

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